Transition-Edge Sensors (TES) are radiation detectors working at cryogenic temperatures [1,2] (~ 100 mK) having capability for sensing very small amounts of energy coming from X-rays (few keV), for example, with superior sensitivity (~ 1 eV). TES can even detect single photons and measure their energy with high accuracy. TES are used in Astrophysics and Cosmology applications, as well as in Nanotechnology and Quantum Technologies. TES have already been implemented on telescopes and in a future (2023) the detector of the high spectral resolution instrument of the next telescope of X-rays from the European Spatial Agency (ESA), Athena, will be constituted by TES [3]. TES are microcalorimeters (electrothermal devices) made of a superconducting (SC) thin film (or bilayer). They take advantage of the steep resistive transition of the SC material between the normal and superconducting states (typically a few mK). This is what makes TES very sensitive to incoming radiation.

TES performances (their spectral energy resolution and response time) depend on TES parameters, which are extracted from fits to the measured complex impedance $Z(o)$; these fits require an electrothermal model of the TES, that is, knowing the number of relevant thermal blocks and their configuration. Therefore, in order to optimize TES performances and improve them, electrothermal modeling plays a key role. Usually, TES parameters are extracted by using the simplest electrothermal model, that is, considering the TES constituted by a single thermal block (1 TB). This is, though, an approximation even when no absorber is present. In this work we develop fits to $Z(o)$ of bare TES considering different configurations with 2 thermal blocks (2 TBs) and analyze when the second TB becomes relevant, and what is its impact on basic TES parameters. We report on the results obtained so far, including a critical analysis of fits reliability and the TES size effects on the TES thermal parameters, which in the end should help us to identify the present TBs.

TES has an extra heat capacity when no absorber is present. In this work we develop fits to $Z(o)$ of bare TES considering different configurations with 2 thermal blocks (2 TBs) and analyze when the second TB becomes relevant, and what is its impact on basic TES parameters. We report on the results obtained so far, including a critical analysis of fits reliability and the TES size effects on the TES thermal parameters.

**TES PHYSICS and OPERATION**

- TESs have an absorber to collect photons and convert the radiation energy into thermal energy (thermalization process) to be sensed by the superconducting part of TES (it heats).
- TES is in thermal contact with a bath, at a cryogenic temperature ($T_{bath}$) through a thermal link (a membrane for X-rays).
- TES resistance increases when a photon is absorbed, and since voltage is kept constant, that produces an inverse peak of current (this is what we measure).
- Operation mode: negative electrothermal feedback [1]. TES is voltage-biased and thus self-heated above $T_{bath}$.
- At low temperatures and large bias, TES is biased above $T_{bath}$ and is self-heated above $T_{bath}$.
- Therefore, TES resistance increases when a photon is absorbed, and the peak of current (what we measure).

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**TES PERFORMANCES:**

- Spectral energy resolution ($\delta E_{FWHM}$).
- Response time ($T_{res}$).

**TES parameters:**

- (1) $\alpha$ and $\beta$; logarithmic derivatives of TES resistance to temperature and current ($C$ and $G$ are TES heat capacity and thermal conductance).
- $\delta E_{FWHM} = 2\sqrt{2ln2} \frac{\alpha - 1}{\sigma} \frac{T_{bath}}{\frac{C}{\sigma} + \frac{G}{\sigma}}$

**RESULTS:**

- $\alpha$, $\beta$, $C$, and $G$.

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**SUMMARY**

<table>
<thead>
<tr>
<th>1TB fits at 70%R_b</th>
<th>2TB fits at 70%R_b</th>
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</thead>
<tbody>
<tr>
<td>Fitted parameters</td>
<td>6 (4) TES parameters when 2 TB (1 TB) is used: $\alpha$, $\beta$, 2 $C$s, 2 $G$s</td>
</tr>
<tr>
<td>Block configuration</td>
<td>Additional hypothesis required</td>
</tr>
</tbody>
</table>

- The fitting procedures for $Z(o)$ using two thermal blocks have been developed.
- The impact of:
  - (1) several fitting algorithms
  - (2) initial values of fitting parameters on the fits quality and fitting parameters reliability have been evaluated.
- The relevance of the 2nd TB has been analyzed as a function of bias and TES size. It turns out that it is increasingly important as bias and TES size increases.
- Two block configurations (hanging and intermediate) have been analyzed. The effects on TES parameters of these configurations and of their required hypotheses have been studied.
- The impact of the 2nd block on the logarithmic sensitivities $\alpha$ and $\beta$ has been analyzed.
- TES thermal parameters ($C$ and $G$) have been analyzed for the different configurations as a function of TES size; some preliminary conclusions have been drawn but more work is required to ascertain the suitable block configuration and identify the blocks.

**REFERENCES**